



Forecasting Air Passenger Traffic Flows in Canada: An Evaluation of Time Series Models and Combination Methods

Mémoire

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Résumé

Ces quinze dernières années, le transport aérien a connu une expansion sans précédent au Canada. Cette étude fournit des prévisions de court et moyen terme du nombre de passagers embarqués/débarqués au Canada en utilisant divers modèles de séries chronologiques : la régression harmonique, le lissage exponentiel de Holt-Winters et les approches dynamiques ARIMA et SARIMA. De plus, elle examine si la combinaison des prévisions issues de ces modèles permet d'obtenir une meilleure performance prévisionnelle. Cette dernière partie de l'étude se fait à l'aide de deux techniques de combinaison : la moyenne simple et la méthode de variance-covariance. Nos résultats indiquent que les modèles étudiés offrent tous une bonne performance prévisionnelle, avec des indicateurs MAPE et RMSPE inférieurs à 10% en général. De plus, ils capturent adéquatement les principales caractéristiques statistiques des séries de passagers. Les prévisions issues de la combinaison des prévisions des modèles particuliers sont toujours plus précises que celles du modèle individuel le moins performant. Les prévisions combinées se révèlent parfois plus précises que les meilleures prévisions obtenues à partir d'un seul modèle. Ces résultats devraient inciter le gouvernement canadien, les autorités aéroportuaires et les compagnies aériennes opérant au Canada à utiliser des combinaisons de prévisions pour mieux anticiper l'évolution du trafic de passager à court et moyen terme.

Mots-Clés : Passagers aériens, Combinaisons de prévisions, Séries temporelles, ARIMA, SARIMA, Canada.

Abstract

This master's thesis studies the Canadian air transportation sector, which has experienced significant growth over the past fifteen years. It provides short and medium term forecasts of the number of enplaned/deplaned air passengers in Canada for three geographical subdivisions of the market: domestic, transborder (US) and international flights. It uses various time series forecasting models: harmonic regression, Holt-Winters exponential smoothing, autoregressive-integrated-moving average (**ARIMA**) and seasonal autoregressive-integrated-moving average (**SARIMA**) regressions. In addition, it examines whether or not combining forecasts from each single model helps to improve forecasting accuracy. This last part of the study is done by applying two forecasting combination techniques: simple averaging and a variety of variance-covariance methods. Our results indicate that all models provide accurate forecasts, with MAPE and RMSPE scores below 10% on average. All adequately capture the main statistical characteristics of the Canadian air passenger series. Furthermore, combined forecasts from the single models always outperform those obtained from the single worst model. In some instances, they even dominate the forecasts from the single best model. Finally, these results should encourage the Canadian government, air transport authorities, and the airlines operating in Canada to use combination techniques to improve their short and medium term forecasts of passenger flows.

Key Words: Air passengers, Forecast combinations, Time Series, ARIMA, SARIMA, Canada.

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Foreword

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Introduction

This thesis studies the forecasting of air passenger volumes in Canada. Its main objective is to evaluate the accuracy of short and medium term monthly forecasts of enplaned/deplaned air passengers in Canada for three different types of flights: domestic, transborder and international.

Transportation has long been recognized as being an important sector of the Canadian economy. While it is true that air transportation greatly depends on the economic activity, the number of enplaned/deplaned air passengers in Canada has displayed a remarkable growth between 2000 and 2010. During this period, the number of enplaned/deplaned air passengers in Canada grew from 52 million up to 66 million in the domestic sector (+26%), from 21 to 22 million in the transborder sector (+5%) and from 13 to 21 million in the international sector (+62%)¹. Total Canadian GDP² has roughly grown from 11 to 13 thousand billion USD (+ 18%) over the same period. Predicting future air passenger volumes is important, as it allows air transport authorities to adapt airport infrastructure to future needs and offers airline companies the capacity to match the increasing passenger demand for air transportation. Furthermore, the air transportation industry is an important employer in Canada, and thus contributes both directly and indirectly to a prosperous Canadian economy. For example, in 2009, there were 92 400 on site employees working in the air transportation sector, compared to 31 700 in the rail industry (Transport Canada, 2011).

This study provides forecasts of air passengers in Canada based on various time series forecasting models. Given that air transportation demand is highly linked to the economy, which is typically characterised by business cycles, or alternations between periods of economic growth and downturns, our data series should exhibit cyclical patterns or seasonality. In addition, any economy is highly susceptible to a variety of shocks of differing nature (political, economic, climatic, etc), which are likely to modify the past trends and the volatility in the data. The majority of the time series models used in this study take both of these characteristics into account. The models tested are: the harmonic regression model, the Holt-Winters exponential smoothing model as well as two ARMA time series models. From the latter class of models, we concentrate on the autoregressive-integrated-moving average model (**ARIMA**) and the seasonal autoregressive-integrated-moving average model (**SARIMA**). Harmonic regression models are estimated by ordinary least squares and produce forecasts based on a

¹These series have been kindly provided by Transport Canada.

²Figures at constant US dollars of year 2000 are available from the Conference Board of Canada.

deterministic function which includes trigonometric terms (Cowpertwait and Metcalfe, 2009). Exponential smoothing models produce forecasts by weighing current and past observations (Bourbonnais and Terraza, 2004). Our ARMA specifications rely on dynamic equations based on the current and past values of air passengers, as well as the current and past values of an error term. Most models take into account the effects of past deterministic shocks. As noted by Emiray and Rodriguez (2003), all these models assume that the future behavior of our air passenger series will be similar to their past behavior. In the short and medium run, this assumption seems appropriate, given that airports are heavy structures characterised by inertia. In addition, we combine the forecasts obtained from the individual models and examine, whether or not, the resulting forecasts are more accurate than their individual counterparts. Two combination techniques are used: *simple averaging* and *variance-covariance weighing schemes*.

There are a great amount of studies, particularly since the 1990's that use time series models to forecast air passenger numbers. The forecasting performance of each model varies depending on the origin country of the passengers, the type of flight considered (domestic, transborder, international), the performance measure and the forecasting horizon (Emiray and Rodriguez, 2003; Oh and Morzuch, 2005; Chu, 2009). No methodological approach has been found to always dominate another in terms of forecasting performance out-of-sample (Shen et al., 2011).

In recent years, combination forecasting techniques have become increasingly popular in the forecasting literature as a means to improve forecasting performance and to control for the uncertainty of relying exclusively on a single model. In the tourism forecasting literature, single model combinations are generally found to outperform the specific models being combined, independently of the time horizon considered (Shen et al., 2008; Coshall, 2009; Shen et al., 2011). However, these results are sensitive to the combination technique (Wong et al., 2007; Coshall, 2009) as well as to region/country-specific characteristics (Wong et al., 2007). Finally, the best performance is likely to be achieved by combining two or three single forecasts at most (Wong et al., 2007).

This study complements the existing literature by providing the following contributions:

1. It deals with recent Canadian air passenger series and provides monthly forecasts over the 2009 to 2010 period.
2. It applies, for the very first time, a wide array of forecast combination techniques to Canadian air passenger data and examines, whether or not, the conclusions of the other papers regarding the performance of these techniques are still valid in this particular case.
3. Finally, this is the only study that uses a Holt-Winters exponential smoothing model in order to forecast the number of monthly air passengers in Canada.

Our results indicate that all of the specific models offer adequate forecasts with MAPE and RMSPE below 10% in most cases. Our forecasts capture the main characteristics of our air passenger series.

ARIMA and SARIMA models usually outperform the harmonic regression and Holt-Winters exponential smoothing models. In addition, the performance of each model depends on the geographical sector considered. Domestic series are better predicted with ARIMA regressions while transborder series are better predicted with SARIMA specifications. On the other hand, international series are better predicted with Holt-Winters. These results stress the fact that no single model dominates the others for all market segments.

Regarding the forecast combinations, we notice that they are always more accurate than the predictions obtained from the single worst model. Furthermore, forecast combinations sometimes dominate those from the single best model. Finally, we note that the performance of combining forecasts is sensitive to the models being combined, the flight sector considered as well as the combination method applied. Simple average appears to provide the poorest performance.

The rest of this master's thesis is structured as follows. The first section reviews the existing literature. The second section describes the empirical time series models as well as the two forecast combination techniques that we employ. The third section presents the data. Finally, the fourth section shows the results of the study whereas the last section provides some conclusions.

Chapter 1

Review of literature

1.1 Univariate models

There is a large amount of literature regarding tourism and air passenger forecasting with univariate time series and smoothing techniques. Most published studies tend to concentrate on three specific regions: the United States, Europe and the Asia Pacific region (Oh and Morzuch, 2005; Lim and McAleer, 2002; Andreoni and Postorino, 2006; Coshall, 2006; Chen et al., 2009). To our knowledge, the lone study on Canada is the work of Emiray and Rodriguez (2003). These authors provide monthly forecasts of enplaned/deplaned air passengers for three market segments (domestic, international and transborder flights), based on data covering the period ranging from January 1984 to September 2002. They consider six time series models (AR(p), AR(p) with seasonal unit roots, SARIMA, periodic autoregressive model (PAR), structural time series model (STSM) and the seasonal unit roots model) as well as the simple average combination method. They conclude that forecasting performance depends on two key elements: the market segment considered and the forecasting horizon. Emiray and Rodriguez (2003) also show that short memory models are better for short term forecasting whereas long memory models are better for long term forecasting. Their paper did not consider smoothing forecasting techniques.

ARIMA and SARIMA models are among the most widely used in the air passenger forecasting literature. According to Zhang (2003), their popularity comes from the fact that they are based on very few assumptions. Furthermore, they are easy to specify with the recursive Box-Jenkins methodology.

Many studies have examined the performance of ARIMA and SARIMA models for predicting air traffic flows. For example, Coshall (2006) forecasts air travel from the United Kingdom to twenty destinations using quarterly data of UK outbound air travelers with several models, which include: the Naive 1 model, the Naive 2 model¹, the Holt-Winters model and a variety of ARIMA models. The Root Mean Squared Error (RMSE) suggests that the ARIMA model outperforms the other models

¹The forecasts in the Naive 1 model for quarterly data are given by: $y_t = y_{t-4}$. In the Naive 2 model, they are given by $y_t = y_{t-4} + [1 + \frac{y_{t-4} + y_{t-8}}{y_{t-8}}]$. For more details, see Coshall (2006).

for all destinations except one. This is true for both additive and multiplicative seasonality models. Andreoni and Postorino (2006) use the yearly data of planned/enplaned passengers at Reggio Calabria airport in the South of Italy to forecast air transport demand. Two univariate ARIMA models and a multivariate ARIMAX model with two explanatory variables (mainly per capita income and the number of movements both to and from Reggio Calabria airport) are used to generate forecasts. The authors conclude that all three models offer accurate forecasts. Kulendran and Witt (2003) generate one, four and six quarter ahead forecasts of international business passengers to Australia from the following four countries: Japan, New Zealand, the United Kingdom and the United States. They consider various forecasting models: the error correction model (ECM), the structural time series model (STSM), the basic structural model (BSM), no change models as well as various ARIMA models. They conclude that forecasting performance varies with the forecasting horizon and depends on the adequate detection of seasonal unit roots. Consequently, ARIMA and BSM models are the most accurate for short term forecasting (one-quarter ahead) whereas the seasonal no change model outperforms the other models for medium term forecasting (four and six quarters ahead).

Chen et al. (2009) estimate monthly arrivals to Taiwan from Japan, Hong Kong and the United States as well as the total amount of monthly inbound air travel arrivals. Interestingly, they divide air arrivals in three categories according to travel purpose: tourism, non-tourism and any purpose. They consider the following forecasting models: Holt-Winters, SARIMA and a Grey forecasting model. The authors conclude that the SARIMA model outperforms all other models when it comes to estimating tourism related arrivals to Taiwan. For all purpose and non-tourism arrivals, the SARIMA model outperforms the other two models for arrivals from Japan and the United States but not for those from Hong Kong or for total arrivals (however, it remains very performant even in these cases). Tsui et al. (2011) forecast air passenger traffic at Hong Kong International Airport using a univariate seasonal ARIMA model and a multivariate ARIMAX model with exogenous explanatory variables. Forecasts are obtained for the period ranging from March 2011 to December 2015. The authors show that the ARIMA specification generates more accurate forecasts than the ARIMAX one over one to three month horizons. In addition, one and two month ahead forecasts are more accurate than three month ones.

Lim and McAleer (2001) predict monthly tourist arrivals to Australia from three destinations: Hong Kong, Malaysia and Singapore. They use the following forecasting models: the single exponential smoothing model, Brown's double exponential smoothing model, the additive and multiplicative seasonal Holt-Winters models as well as the non-seasonal exponential smoothing model. Their performance is evaluated with the RMSE (Root Mean Squared Error) criterion. They find that for the series in levels, the multiplicative Holt-Winters model offers the best performance for Hong Kong and Singapore whereas in the case of Malaysia, the additive Holt-Winters model is the most accurate. Cho (2003) forecasts tourist arrivals to Hong Kong from: the United Kingdom, Japan, Korea, Singapore and Taiwan. He uses three different forecasting models: multiplicative exponential smoothing model, ARIMA and artificial neural network model (ANN). He finds, that according to both the MAPE (Mean

Absolute Percentage Error) and the RMSE (Root Mean Square Error), the ANN model outperforms ARIMA and Holt-Winters for forecasting tourist arrivals to Hong Kong from all of the aforementioned countries with the exception of the United Kingdom. For the latter, Holt-Winters offers the best performance. However, the author also notes that ARIMA and Holt-Winters usually give a good forecasting performance with a MAPE inferior or equal to 20%.

The preceding studies suggest that the forecasting performance of ARIMA and SARIMA models varies with the market segment considered, the forecasting horizon as well as the origin and destination countries of the passengers. In addition, these models are generally found to offer accurate forecasts in the short and medium term.

1.2 Forecast combinations

Forecasting combination techniques offer an alternative approach to single models' forecasts. Bates and Granger (1969) were the first to propose such techniques to improve the forecasting accuracy of individual models (Wong et al., 2007). Over the last three decades, these techniques have become highly prevalent in the forecasting literature. They have been applied with success to numerous fields, including: macroeconomics (Poncela and Senra, 2006; Bjørnland et al., 2011), meteorology (Brown and Murphy, 1996), banking (Chan et al., 1999) and tourism (Shen et al., 2008, 2011; Coshall, 2009).

Many authors have outlined the reasons behind the prevalence of these techniques. For example, Timmermann (2006) points out that combined forecasts allow to better aggregate all relevant information captured in different single model forecasts and they are more robust against a misspecification of the data generating process. Therefore, combined forecasts can improve the forecasting accuracy and they are seen as being more comprehensive (see also Bunn, 1988). Brown and Murphy (1996) note that combination forecasts are more likely to improve forecasting performance when each single model forecast being combined is independent of the other (or uncorrelated). Timmermann (2006) also stresses that combination forecasts are particularly useful when structural breaks are present in the data series. Again, each individual model will process differently the structural breaks. The time series used in this study are very likely to exhibit structural breaks, given the events that took place during the time period under scrutiny (September 11th terrorist attacks, the Gulf war, the 2009 financial crash).

In this paper, we will apply two combination techniques that have been widely used in the forecasting literature: simple averaging and variance covariance methodologies. Both techniques take a weighted average of single model forecasts. However, they differ in how weights are computed. The simple average method assigns equal weights to each predicted value from the single models that are being combined. These weights are straightforward to compute : they are equal to the inverse of the number of single model forecasts being combined. Various versions of the variance-covariance method exist (see Bates and Granger (1969), Coshall (2009) and Chu (1998)). All of them assign unequal weights to each single model forecast being combined and take into consideration the past performance of the

forecasting model.

The conclusions that have been reached regarding these combination techniques vary from one paper to another. For example, [Winkler and Makridakis \(1983\)](#) generate forecasts for 1001 economic time series with different types of data, and conclude that more complex combination methods slightly outperform the simple average method for long term forecasting horizons. In the air transportation forecasting literature, [Chu \(1998\)](#) provides monthly forecasts of tourist arrivals to Singapore for year 1988 using a SARIMA and a sine wave regression model. He applies a version of the variance-covariance method adapted for seasonal data. Forecasting performance is evaluated using the MAPE. He finds that the combined forecast is more accurate than the ones issued from ARIMA and sine wave. [Shen et al. \(2011\)](#) use tourist flows from the United Kingdom to seven major touristic destinations to point out that unequal weighing schemes outperform the simple average method. In contrast, [Coshall \(2009\)](#) studies tourist departures from the United Kingdom to twelve destinations. He concludes that the performance of different combination methods depends on the forecasting horizon. In this particular case, the variance-covariance method outperforms simple averaging for one and two years ahead forecasts while the reverse is true for three years ahead forecasts. Finally, [Wong et al. \(2007\)](#) study tourist arrivals to Hong Kong. They find that forecasting performance depends on the number of single model forecasts being combined. Thus, they mention that the best performance is likely to be achieved by combining two or three single model forecasts at most.

In sum, most papers on combination forecasts highlight that forecasts based on some averaging of individual models' forecasts are more accurate than those based on a particular model. Moreover, unequal weighting schemes, which account for the past prediction performance of the individual models, seem in general more appropriate than weighting equally the forecasts of the individual models.

1.3 Alternative forecasting methods

Univariate time series linear models are only one of the many methods used for forecasting air passenger demand. Over the years, many other promising alternatives have been developed. A very brief overview of some of these methods is presented in this section.

One of the major limitations of ARIMA models is their arbitrary parametric functional shape. A priori specifications may fail to capture important non-linearities and interactions which have not been explicitly modeled. These can be captured by **artificial neural network models (ANN's)** but at the expense of interpretability. The contribution of each regressor cannot be interpreted individually.

As an example, [Chen et al. \(2012\)](#) used **back propagation neural networks** to identify the factors that influence air passenger and air cargo demand from Japan to Taiwan. They found that air transport and air cargo demand are generally influenced by different factors but that there are some common factors which influence both. This allowed them to construct models which possess very high forecasting accuracy in the short and medium term. For example, their air passenger demand model had a MAPE

(Mean Absolute Percentage Error) of 0.34%. However, they noted that the performance of neural network models heavily depends on choosing an appropriate training set.

Bao et al. (2012) compare the forecasting performance of a Holt-Winters exponential smoothing model, a univariate time series model (ARIMA) and the following **support vector machines based models** : single Support Vector Machines (SVM's), Ensemble Empirical Mode Decomposition based Support Vector Machines (EEMD-SVM's) and Ensemble Empirical Mode Decomposition Slope based method Support Vector Machines (EEMD-Slope-SVM's). They do this using the monthly air passenger data from six American and British airline companies and the following performance criteria: Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE), Geometric Mean Relative Absolute Error (GMRAE) and the directional statistic (DS). They conclude the following: (i) single SVM's outperform ARIMA and Holt-Winters, (ii) EEMD-SVM's outperform single SVM's and (iii) EEMD-Slope-SVM's are more accurate than EEMD-SVM's (and therefore also outperform ARIMA, Holt-Winters and single SVM's).

Fildes et al. (2011) make use of a wide variety of **multivariate models** to study air traffic flows between the United Kingdom and five other countries: Germany, Sweden, Italy, the USA and Canada. They use the following econometric models: an autoregressive distributed lag model (ADL), a pooled ADL model, a time-varying parameter model (TVP) as well as an automatic method for econometric model specification. They also consider the previous four models augmented by a world trade explanatory variable (which measures the total trade of all industrial countries). In addition, they apply the following (mostly univariate) models: a vector autoregressive model (VAR), a vector autoregressive model with the world trade variable, an exponential smoothing model, an autoregressive model of order three (AR(3)) as well as Naive I and Naive II benchmark models. Forecasting performance is evaluated according to the following four criteria: (i) Root Mean Square (Percentage) Error (RMSE), (ii) Geometric Root Mean Square Error (GRMSE), (iii) Mean Absolute Scaled Error (MASE), and (iv) Geometric Mean Relative Absolute Error (GRelAE). They find that ADL models with the inclusion of a world trade variable outperform overall the univariate models (exponential smoothing and AR(3) models) but that the difference in forecasting performance is usually small, although it varies depending on the forecasting performance criterion used (usually larger when using RMSE).

Grosche et al. (2007) develop two **gravity models** in order to forecast air passengers between city pairs. Both models include mostly geoeconomic variables. The first model excludes city-pairs which involve multi-airport cities. Hence, it excludes competition. It uses such variables as: population, distance between airports, average travel time and buying power index to predict travel demand. On the other hand, the second model includes multi-airport cities as well as variables that take them into account (such as the number of competing airports, the average distance to competing airports). Both models are found to be statistically valid and fit the data well.

Chapter 2

Econometric Methodology

Our goal in this thesis is to use the most up-to-date monthly data available at Transport Canada in order to evaluate the accuracy of both short run and medium run forecasts for the number of enplaned/deplaned air passengers in Canada. We consider the most popular models for forecasting air traffic flows: a harmonic regression, an exponential smoothing model as well as two standard ARMA-based time series models. In addition, we examine whether or not combining the forecasts of the above models helps to build more accurate forecasts. All our investigations are carried out using the R statistical software¹.

2.1 Harmonic regression model

Harmonic regression² is a linear specification that accounts for seasonal patterns with the help of two trigonometric functions: sine and cosine. Note that in this model both the linear time trend and the seasonal pattern are considered as being fixed and stable over time (deterministic). The time series can be represented as:

$$y_t = \alpha + \beta t + \sum_{j=1}^{\lfloor s/2 \rfloor} (\gamma_j \cos \lambda_j t + \gamma_j^* \sin \lambda_j t) + \sum_{j=1}^k D_{t,j} + \varepsilon_t \quad (2.1a)$$

with:

$$\lambda_j = \frac{2\pi j}{s} \quad j = 1, \dots, \lfloor s/2 \rfloor. \quad (2.2a)$$

In the preceding equations, α is a constant, β is the linear time-trend coefficient, γ_j and γ_j^* are parameters linked to the seasonality components, λ_j are the harmonics of order j , s represents the number of seasons, $\lfloor \cdot \rfloor$ is the integer function³ and $D_{t,j}$ stands for exogenous variables which capture shocks

¹For a complete description of this software see R Development Core Team (2012).

²This section relies on Harvey (1993, p.138 and 186-187) and Cowpertwait and Metcalfe (2009, Ch. 5.6 to 5.10).

³The nearest integer function $\lfloor s/2 \rfloor$ implies that $\frac{s}{2} = s$ if s is even and $\frac{s}{2} = \frac{s-1}{2}$ if s is odd.

to the series. In addition, ε_t is assumed to be a white noise error term, i.e, a normally, independantly and identically distributed random variable such that ⁴:

$$E(\varepsilon_t) = 0 \quad \forall t \quad (2.3a)$$

$$E(\varepsilon_t^2) = \sigma_\varepsilon^2 \quad \forall t \quad (2.3b)$$

$$Cov(\varepsilon_t, \varepsilon_s) = 0 \quad \forall t, \forall s, t \neq s. \quad (2.3c)$$

The model that best fits the data is selected through the backward selection procedure⁵. This procedure is implemented in the function `step()` of the `stats` package in R. It starts by considering the model with all of the variables. From this model, the variables that lead to the biggest decrease in Akaike's information criterion (AIC) are eliminated one by one until the AIC cannot be further lowered. Consequently, the best fitting model is the one that minimizes the AIC. The Breush-Pagan and the Durbin-Watson tests⁶ are then applied to check whether the assumption of homoscedastic and uncorrelated residuals holds.

The Breush-Pagan test is implemented with the `bptest()` function of the `lmtest` package⁷ in R. Note that this function applies a version of the test which is robust to departures from normality. Regarding the Durbin-Watson test, we use the `dwtest()` function of the aforementioned package. If the null hypothesis of either or both of these tests is rejected, the ordinary least squares estimator will not be efficient and the `vcovHAC()`⁸ function of the `sandwich` package in R is used to produce a covariance matrix that is heteroscedasticity and autocorrelation consistent. Once this is achieved, forecasts are generated using the `predict()` function of the `stats` package. Finally, it is important to note that the harmonic regression model can also contain dummy variables that account for structural shocks that occurred during the period under investigation.

2.2 Holt-Winters exponential smoothing model

The Holt-Winters⁹ recursive procedure is an exponential smoothing technique that was developed as a means of forecasting time series exhibiting both a trend and a seasonal pattern.

There exist two versions of this procedure: additive and multiplicative. In both versions, forecasts will depend on the following three components of a seasonal time series: its level, its trend and its seasonal coefficient. In addition, both are implemented in the `HoltWinters()` function of the `forecast` package¹⁰ in R. The version to be used depends on the type of seasonality, additive or multiplicative,

⁴See Greene (2005, Ch.20.2).

⁵Cornillon and Matzner-Løber (2007, p.167).

⁶The description of these tests is inspired from Greene (2005, Ch.10 and 12).

⁷For a complete description of this package, see Zeileis and Hothorn (2002).

⁸For details, see Zeileis (2004).

⁹This section follows Bourbonnais and Terraza (2004, Ch.2.II.C), Montgomery et al. (2008, p. 210-216) as well as Chatfield (2003, p. 20, 78-79).

¹⁰See Rob J Hyndman et al. (2013).

that we observe in the data by plotting the series with respect to time. Thus, the additive version ought to be considered whenever the seasonal pattern of a series has a constant amplitude over time (Kalekar, 2004). In such a case, the series can be written as:

$$y_t = a_t + b_t t + S_t + \varepsilon_t \quad \text{with} \quad \sum_{t=1}^{t=s} S_t = 0. \quad (2.4)$$

In the preceding equation, a_t represents the level of the series at time t , b_t the slope of the series at time t , S_t the seasonal coefficient of the series at time t and s the periodicity of the series. Furthermore, ε_t are error terms with mean 0 and constant variance.

Inversely, when a series displays a seasonal pattern characterised by an amplitude that varies with the level of the series, the multiplicative version is a better choice (Kalekar, 2004; Lim and McAleer, 2001). In such a case, the series can be represented as follows:

$$y_t = (a_t + b_t t) S_t \varepsilon_t \quad \text{with} \quad \sum_{t=1}^{t=s} S_t = s. \quad (2.5)$$

In the preceding equation¹¹, all the elements are defined as previously. In what follows, we will concentrate on the multiplicative case. In the latter case, forecasts can be obtained using the following equation:

$$\hat{y}_{t+h} = (a_t + h b_t) S_{t-12+1+(h-1) \bmod 12} \quad (2.6a)$$

where h represents the forecasting horizon and where a_t , b_t and S_t need to be estimated using the following three equations:

$$a_t = \alpha \left(\frac{y_t}{S_{t-12}} \right) + (1 - \alpha)(a_{t-1} + b_{t-1}) \quad (2.7a)$$

$$b_t = \beta (a_t - a_{t-1}) + (1 - \beta) b_{t-1} \quad (2.7b)$$

$$S_t = \gamma \left(\frac{y_t}{a_t} \right) + (1 - \gamma) S_{t-12}. \quad (2.7c)$$

In these equations, α , β and γ are smoothing parameters (or weights) for the level, the trend and the seasonal component respectively. All three parameters lie between 0 and 1 and can be interpreted as discounting factors: the closer to 1, the larger the weight of the recent past (Coshall, 2009). Cho (2003) also mentions that the closer the parameter is to 0, the more the related component is constant over time. Optimal values for the three parameters are obtained by minimizing the squared one-step ahead forecast errors¹² (Coshall, 2009).

Finally, it is important to note that the Holt-Winters procedure has been widely used in the forecasting literature for over fifty years. Its popularity stems from the fact that it is simple to apply. Furthermore, it does not entail fitting a parametric model (Gelper et al., 2010). However, it has the disadvantage of being sensitive to structural breaks (Gelper et al., 2010), which are not accounted for.

¹¹The multiplicative seasonality model can also be expressed as: $y_t = (a_t + b_t t) S_t + \varepsilon_t$.

¹²The forecast error is the difference between the forecasted and the observed value of a series.

2.3 Univariate time series models

2.3.1 Autoregressive-moving average model

The two univariate time series models used in this paper are based on the autoregressive-moving average or ARMA(p,q) model¹³. The idea behind this model is that the value taken by a time series at a given time t , denoted y_t , depends on two additive terms: (i) the past of the time series (an autoregressive component of order p) and (ii) the past of the disturbances of the data generation process (a moving average component of order q). Hence, the general form of an ARMA model is:

$$y_t = \sum_{i=1}^p \phi_i y_{t-i} - \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (2.8)$$

where the ϕ_i 's ($i=1, \dots, p$) are called autoregressive parameters, the θ_i 's ($i=1, \dots, q$) are called moving average parameters and the ε 's are white noise error terms. Note that ARMA models are often expressed in a more elegant and compact form. Let's first introduce the three following notations:

$$B^m y_t = y_{t-m} \quad (2.9a)$$

$$\phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p \quad (2.9b)$$

$$\theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q. \quad (2.9c)$$

B represents the backward shift operator, $\phi(B)$ is the autoregressive part polynomial of order p and $\theta(B)$ is the moving average part polynomial of order q . Using equations (2.9a) to (2.9c), we rewrite the ARMA model as follows:

$$\phi(B)y_t = \theta(B)\varepsilon_t. \quad (2.10)$$

An important assumption imposed on ARMA models is that of stationarity of the time series. A time series y_t is (weakly or covariance) stationary if the three following conditions are met¹⁴:

$$E(y_t) = \mu \quad \text{and} \quad E(y_t^2) = \mu_2' \quad \forall t \quad (2.11a)$$

$$\text{Var}(y_t) = \mu_2' - \mu^2 = \sigma_y^2 = \gamma(0) \quad \forall t \quad (2.11b)$$

$$\text{Cov}(y_t, y_s) = E[y_t y_s] - \mu^2 = \gamma(|t-s|) \quad \forall t, \forall s, t \neq s. \quad (2.11c)$$

Equations (2.11a) and (2.11b) tell us that the mean and the variance of y_t must be independent of time. This implies that any shock on y_t will be of temporary nature and will not permanently influence the future values of the series (Rodrigues and Osborn, 1999). Equation (2.11c) states that the covariance between any two observations y_t and y_s must be independent of origin times t and s . Indeed, it must only be a function of the time difference ($t-s$) between the two observations.

Many economic time series display either deterministic or stochastic non-stationarity¹⁵. In addition, some time series may exhibit both types. The first type of non-stationarity is due to the presence

¹³The description of this model follows Harvey (1993, Ch.2.1) and Bourbonnais and Terraza (2004, p.82).

¹⁴As given in Bresson and Pirotte (1995, p.19).

¹⁵The following section is based on Bourbonnais and Terraza (2004, p.133-139), Harvey (1993, p.20) and Greene (2005, p.597).

of a deterministic trend. Series exhibiting deterministic non-stationarity are said to follow a trend-stationary process. To render them stationary, one can exploit the differences between the observations and the deterministic trend. The general form of a purely trend-stationary series is:

$$y_t = f_t + \varepsilon_t \quad (2.12)$$

where f_t is a linear or non-linear function of time and ε_t is a stationary process.

The second type of non-stationarity is called stochastic non-stationarity. Starting with the characteristic equation of model (2.10):

$$\phi(B) = 0 \quad (2.13)$$

we say that equation (2.10) has a unit root if at least one of the roots of equation (2.13) is equal to one (unit root). The presence of a unit root in process (2.10) generates stochastic non-stationarity. Such a process can be rendered stationary through differencing. The same holds true whenever at least one of the roots of the characteristic equation is inferior to one (lies inside the unit circle). Inversely, if all the roots of the characteristic equation are superior to one (lie outside the unit root circle), then the series is stationary.

It is important to determine the appropriate transformation required to render a time series stationary. Indeed, applying the wrong transformation can significantly alter the statistical properties of the series, thus producing biased forecasts. For example, using departures from a deterministic trend for a time series exhibiting stochastic non-stationarity is likely to create spurious correlation in the residuals of an ARMA model¹⁶.

There exist various procedures for determining the type of non-stationarity exhibited by a time series. In this thesis, we will use an Augmented Dickey-Fuller test procedure, that is part of the Box-Jenkins methodology described in section 2.3.4. This leads us to the ARIMA model.

2.3.2 Autoregressive-integrated-moving average model

The autoregressive-integrated-moving average¹⁷, or ARIMA(p,d,q) model differs from the ARMA(p,q) model in one significant way: it contains a d parameter. This parameter states the level of non-seasonal differencing that is required to render a non-stationary time series stationary (when differencing is appropriate). Once stationarity is achieved, the series can be represented using an ARMA(p,q) model. Note that if the series is already stationary and does not need to be differenced, then it can directly be modeled through an ARMA(p,q). The general form of the ARIMA (p,d,q) model reads

$$\phi(B)(1-B)^d y_t = \theta(B)\varepsilon_t \Leftrightarrow \phi(B)\Delta^d y_t = \theta(B)\varepsilon_t, \quad d \geq 0 \quad (2.14)$$

where $\Delta^d = (1-B)^d$ is the non-seasonal differencing operator. It is important to mention that the ARIMA model is a short memory model (Bourbonnais and Terraza, 2004, p.265). Therefore, it is not

¹⁶See Bourbonnais and Terraza (2004, p.137-139).

¹⁷This section is based on Harvey (1993, Ch.5.2) and Greene (2005, Ch.20.3.1).

able to account for dependance between observations that are far apart over time. Another shortfall of the ARIMA specification (2.14) is that it is unable to account for seasonality and shocks. Monthly patterns can be captured by including dummy regressors of the form:¹⁸

$$D_{jt} = \begin{cases} 1 & \text{if } t = j, j + 12, j + 24 \dots \\ 0 & \text{if } t \neq j, j + 12, j + 24 \dots \\ -1 & \text{if } t = 12, 24, 36 \dots \end{cases}$$

Such seasonality is supposed to remain stable over time.

2.3.3 Seasonal autoregressive-integrated-moving average model

Like the ARIMA model, the seasonal autoregressive-integrated-moving average model¹⁹, or SARIMA (p,d,q) x (P,D,Q)_s, is a short memory model (Bourbonnais and Terraza, 2004, p.265). In addition, it is a more flexible model, given that it accounts for stochastic seasonality. Such seasonality is present when the seasonal pattern of a time series changes over time. In such a case, the time series will contain a seasonal unit root and will need to be seasonally differenced. This will be done through the seasonal difference parameter, "D". If this parameter equals zero, then the seasonal pattern exhibited by the time series is relatively stable over time and can be modeled uniquely through the seasonal autoregressive ($\Phi(B^s)$) and moving average ($\Theta(B^s)$) terms, of order P and Q respectively. These are given by the following equations:

$$\Phi(B^s) = 1 - \Phi_1 B^s - \dots - \Phi_P B^{Ps} \quad (2.15a)$$

$$\Theta(B^s) = 1 - \Theta_1 B^s - \dots - \Theta_Q B^{Qs}. \quad (2.15b)$$

These terms allow us to express the SARIMA model as follows:

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D y_t = \theta(B)\Theta(B^s)\varepsilon_t, \quad d, D \geq 0 \quad (2.16)$$

where s denotes the periodicity of the time series (for example, $s = 12$ for monthly time series).

2.3.4 Box-Jenkins method

A common and effective method for specifying and estimating ARMA-based models is the iterative Box-Jenkins method²⁰. Its advantage is that it can be applied to many types of time series: seasonal, non-seasonal, stationary... It proceeds in four steps: (i) identification, (ii) estimation (iii) diagnostic testing and (iv) forecasting.

Identification

The first step of the Box-Jenkins method is to verify if the time series are stationary and if seasonal patterns need to be modeled. Plotting the time series can provide useful information. In general, series

¹⁸Based on Harvey (1993, p.137).

¹⁹The following section relies on Harvey (1993, p.139-142) and Chen et al. (2009).

²⁰See Bresson and Pirotte (1995, Ch. 2.1 to 2.4).

that cover a large number of years and that exhibit a pronounced seasonal pattern will not have a stable variance. The most common methods for stabilizing the variance are the logarithmic or the Box-Cox transformations. Once the series are variance stationary, it is important to check that their mean is constant through time. This can be done by applying the Augmented Dickey-Fuller or ADF test²¹. This test requires estimating the two following equations:

$$\Delta y_t = \beta_1 + \beta_2 t + \pi y_{t-1} + \sum_{j=1}^k \gamma_j \Delta y_{t-j} + \varepsilon_t \quad (2.17a)$$

$$\Delta y_t = \beta_1 + \pi y_{t-1} + \sum_{j=1}^k \gamma_j \Delta y_{t-j} + \varepsilon_t \quad (2.17b)$$

where: y_t is a time series, Δ is the first difference operator, β_1 is a constant, ε_t is a white noise error term, β_2 is the coefficient of the time trend and k is the lag order of the autoregressive process. The two equations above allow us to determine whether or not a time series contains: (i) a stochastic trend ($\pi = 0$) also called unit root, (ii) a deterministic trend ($\beta_2 \neq 0$) or (iii) both. We answer these questions by applying the following strategy²².

First, equation (2.17a) is used to test the null hypothesis of the presence of a unit root ($H_{\tau_3} : \pi = 0$) with a non-standart t-test. If the null hypothesis is rejected, the data display no stochastic trend and they need no differencing. If the null hypothesis cannot be rejected, the data does contain a unit root and we proceed to testing the joint hypothesis of a unit root and a null deterministic trend ($H_{\phi_2} : \beta_2 = \pi = 0$) with a non-standart F-test²³. If H_{ϕ_2} is rejected, we need to check again that the data include a stochastic trend under the null that the model includes a deterministic trend by comparing the empirical t-value of the unit root coefficient π with standard normal cutoffs. If the stochastic trend is confirmed in the latter test, differencing the residuals from a parametric fit to the time trend needs to be applied. We then check that the latter transformed series is stationary by using equation (2.17b).

We now focus on equation (2.17b). If H_{ϕ_2} cannot be dismissed, the latter equation is used to test the null hypothesis of a unit root ($H_{\tau_2} : \pi = 0$) without trend (with a non-standart t-test). If the null hypothesis cannot be rejected, the series exhibits stochastic non-stationarity and needs to be differenced. In such a case, equation (2.17b) is applied to the series in first-differences to verify that they are stationary and need no further differencing.

Once the ADF test has been applied and the series have been properly modified, we proceed by modeling the seasonality both in a deterministic and a stochastic manner. The ARIMA model (2.14) therefore includes dummies to capture the monthly effects, as outlined in section 2.3.2. Regarding the SARIMA model, the Osborn, Chui, Smith and Birchenhall test (OSCB)²⁴ is used to determine whether the seasonal pattern changes sufficiently over time to warrant seasonal differencing. This test

²¹We follow Pfaff (2008, p.61-63).

²²This is a simplified version of the strategy recommended by Harris (1992) or Pfaff (2008, Ch. 3.2).

²³Note that this tests is based on a data generation process which assumes a unit root with no trend. However, the equation includes the trend variable as a nuisance.

²⁴This section is based on Han and Thury (1997), Osborn et al. (1988) and Rodrigues and Osborn (1999).

is based on the following equation:

$$\Delta^1 \Delta_{12} y_t = \mu_t + \beta_1 \Delta_{12} y_{t-1} + \beta_2 \Delta^1 y_{t-12} + \sum_{j=1}^p \gamma_j \Delta^1 \Delta_{12} y_{t-j} + \varepsilon_t \quad (2.18)$$

where $\Delta^1 = (1-B)$ is the differencing operator and $\Delta_{12} = (1 - B^{12})$ is the seasonal differencing operator. Equation (2.18) allows us to test the null hypothesis that both differencing operators need to be applied to the time series, versus the alternative that only one of them is necessary. This hypothesis is primarily verified with one sided t-tests on β_1 and β_2 . Critical values are obtained through simulation. Whenever the need for Δ^1 differencing has already been established prior to applying the OSCB test, equation (2.18) can be slightly modified in order to test for the presence of seasonal unit roots only :

$$\Delta_{12} z_t = \mu_t + \beta_1 S(B) z_{t-1} + \beta_2 z_{t-12} + \sum_{j=1}^p \gamma_j \Delta_{12} z_{t-j} + \varepsilon_t. \quad (2.19)$$

Note that in the preceding equation, $z_t = \Delta^1 y_t$ and $S(B) = \sum_{i=0}^{11} B^i$. The need for the Δ_{12} filter is established using the overall F_{β_1, β_2} OSCB statistic.

Once the differencing parameters are estimated, one must select the model that best fits the data. This is equivalent to determining the orders of the autoregressive and moving average terms. The selection is based on some information criterion (Akaike's or Schwarz's). The preferred model is the one that minimizes the chosen information criterion. In this thesis, we use Akaike's information criterion²⁵.

Estimation

Once the model has been specified, its autoregressive and moving average parameters need to be estimated (as well as their seasonal equivalents in the case of a SARIMA model). The most commonly used method are non-linear least squares. The estimation is carried out in R with the `auto.arima()` function²⁶ of the forecast package.

Diagnostic checking

The final step of the Box-Jenkins method consists of verifying the statistical validity of the estimated model. The latter is valid if its disturbances are uncorrelated. In this thesis, this is verified through the Ljung-Box portmanteau test²⁷, whose null hypothesis is that there are no residual autocorrelations for h lags. The test statistic is given by:

$$Q = n(n+2) \sum_{k=1}^h \frac{\rho_k^2}{n-k} \quad (2.20)$$

where:

$$\rho_k = \frac{\sum_{t=k+1}^n \hat{\varepsilon}_t \hat{\varepsilon}_{t-k}}{\sum_{t=1}^n \hat{\varepsilon}_t^2}. \quad (2.21)$$

²⁵For more details, see Harvey (1993, p.79).

²⁶For a description of this package, see Hyndman and Khandakar (2007).

²⁷As described in Ljung and Box (1978) or Bourbonnais and Terraza (2004, p.190).

Note that ρ_k is the autocorrelation at lag k ; $\hat{\varepsilon}$ are the residuals of the fitted model; n is the total number of observations and h is the total number of lags considered. The null hypothesis is rejected when Q is superior to the 95th percentile of the χ^2 distribution with h degrees of freedom. The Ljung-Box test is implemented in the `Box.test()` function of the `stats` package in R.

Forecasting with autoregressive-moving average models

Forecasting²⁸ a univariate time series consists of predicting its future values, which are denoted \hat{y}_{t+s} , by using the estimated models. The goal in this thesis is to get the best forecasts with the help of the models described in section 2.3. The expected value of y_t at time $t + s$, given its past realizations, is:

$$E(y_{t+s}|y_t, y_{t-1}, \dots) = \phi_1 E(y_{t+s-1}|y_t, y_{t-1}, \dots) + \dots + \phi_p E(y_{t+s-p}|y_t, y_{t-1}, \dots) \\ + E(\varepsilon_{t+s}|y_t, y_{t-1}, \dots) - \dots - \theta_q E(\varepsilon_{t+s-q}|y_t, y_{t-1}, \dots), \quad s = 1, 2, \dots \quad (2.22)$$

where the expected value of future error terms (ε_{t+j} , $j=1,2,\dots$), conditional on the information at time t , is 0 and the expected value of future values of the series (y_{t+j} , $j=1,2,\dots$), conditional on the information at time t , is given by \hat{y}_{t+j} .

This equation clearly shows that our forecasts s periods ahead will not only depend on the true observations but also on the forecasts from time t to time $t + s - 1$. In addition, after a certain lead time, forecasts might be based only on previous forecasts and not on true observations. The latter are called "bootstrap forecasts" (Bresson and Pirotte, 1995).

Finally, it is important to note that if the logarithmic transformation is applied to stabilize the variance of the time series, then the forecast is in terms of logarithms. Given that our harmonic regression and univariate time series models are based on the assumption that the error term is a white noise, the logarithmic series will follow a lognormal distribution with mean $e^{\frac{1}{2}\sigma^2}$. Consequently, the anti-log forecasts need to be corrected²⁹.

2.4 Forecasting combination techniques

As our goal is to combine forecasts of selected models, this work applies two of the most popular combination methods employed recently in the tourism literature: (i) the *simple average* method and (ii) the *variance-covariance* method. Both if these methods produce a combined forecast for time t , denoted f_{ct} , by taking a weighted average of the single model forecasts. More precisely³⁰:

$$f_{ct} = \sum_{j=1}^J w_{jt} f_{jt} = w_{1t} f_{1t} + w_{2t} f_{2t} + \dots + w_{Jt} f_{Jt} \quad \text{with} \quad \sum_{j=1}^J w_{jt} = 1. \quad (2.23)$$

In this equation, f_{jt} represents the forecast for time t generated by the j^{th} single time series model, J represents the number of single model forecasts being combined and w_{jt} represents the weight

²⁸This section is based on Harvey (1993, Ch.2.6), Bourbonnais and Terraza (2004, p.244-247).

²⁹See Cowpertwait and Metcalfe (2009, Ch. 5.10.1).

³⁰See Wong et al. (2007).

assigned to the j^{th} single model forecast (for time t). The key point when applying combination techniques will be determining the weight that ought to be given to each single model forecast.

Simple average method

As outlined by [Bates and Granger \(1969\)](#), the most straightforward weighing scheme gives the same weight to each single model forecast being combined: $w_{1t} = w_{2t} = \dots = w_{Jt} = \frac{1}{J}$.

Variance-covariance method

The variance-covariance weighing method³¹ is among the first weighing schemes proposed in the literature to optimally combine forecasts from different models. Let's assume that we wish to combine two single model forecasts of yearly frequency. In such a case, the combined forecast (f_{ct}) is given by:

$$f_{ct} = w_{1t}f_{1t} + (1 - w_{1t})f_{2t} \quad \text{with} \quad 0 < w_{1t} < 1. \quad (2.24)$$

If we let y_t denote the current realization of a given time series at time t , the estimated prediction error e_{ct} of the combined forecast is given by:

$$e_{ct} = y_t - f_{ct} = w_{1t}e_{1t} + (1 - w_{1t})e_{2t} \quad (2.25)$$

where e_{jt} , $j=1,2$, denotes the estimated prediction error of each individual model for time t . Denoting the corresponding true error by ε_{jt} and assuming no bias in the estimates, the errors' variance σ_{ct}^2 would be given by:

$$\sigma_{ct}^2 = E(\varepsilon_{ct}^2) = w_{1t}^2\sigma_1^2 + (1 - w_{1t})^2\sigma_2^2 + 2w_{1t}(1 - w_{1t})\rho\sigma_1\sigma_2 \quad (2.26)$$

where σ_{jt}^2 denotes the variance of the j^{th} individual model's prediction error - assumed to be constant over time - and ρ is the related error's correlation. To obtain optimal forecasts, [Bates and Granger \(1969\)](#) propose to determine the weight w_{1t} that minimizes σ_{ct}^2 . They show that this weight is given by :

$$w_{1t}^{min} = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}. \quad (2.27)$$

In practice, σ_1^2 , σ_2^2 and ρ are unknown. Assuming $\rho = 0$, equation (2.27) becomes :

$$w_{1t}^{min} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}. \quad (2.28)$$

[Bates and Granger \(1969\)](#) propose to render the optimal weighing feasible by computing the magnitudes w_{1t} and $(1 - w_{1t})$ with an approach called the **variance-covariance method with shifting window (SWVC)**. In this method, the weights associated with the combined forecast for a given

³¹As described in [Chu \(1998\)](#).

month take into consideration the forecasting performance of the individual models during the m preceding months. This is done through e_{jT} , the sample's forecast error of the j^{th} individual model at time T . In this thesis, we set $m = 12$. The resulting formula is therefore:

$$w_{1t} = \frac{\sum_{T=t-12}^{t-1} e_{2T}^2}{\sum_{T=t-12}^{t-1} e_{1T}^2 + \sum_{T=t-12}^{t-1} e_{2T}^2}. \quad (2.29)$$

Note that the weights change through time.

Chu (1998) proposed to adapt equation (2.29) to seasonal monthly time series. This gave rise to the **seasonal variance-covariance method (SVC)**. In this technique, the weights that are needed to obtain the combination forecast for a given month take into consideration the forecasting performance of each single model forecast during the corresponding month of the previous year. For example, if we want to determine the combined forecast for January 2010, the combination weights will reflect the forecasting performance of the single model forecasts for January 2009. Consequently, the weights can be determined according to the following equation:

$$w_{1t} = \frac{e_{2,t-12}^2}{e_{1,t-12}^2 + e_{2,t-12}^2}. \quad (2.30)$$

Finally, the **variance-covariance method with fixed window (FWVC)**³² differs from the one with shifting window in that the weights reflect the forecasting performance of the specific models for a chosen year³³. Hence, the same set of observations is used to calculate all the weights, resulting in the formula:

$$w_{1t} = \frac{\sum_{T=\tau-12}^{\tau-1} e_{2T}^2}{\sum_{T=\tau-12}^{\tau-1} e_{1T}^2 + \sum_{T=\tau-12}^{\tau-1} e_{2T}^2}. \quad (2.31)$$

In this thesis, we use $\tau = \text{January 2010}$.

2.5 Forecasting performance criteria

In order to compare the predictive accuracy of the various combination methods, we use two forecasting performance criteria: i) MAPE (Mean Absolute Percentage Error) and iii) RMPSE (Root Mean Squared Percentage Error). These two criteria have widely been used in the air passenger and tourism forecasting literatures (Emiray and Rodriguez, 2003; Coshall, 2006; Chen et al., 2009). Given that they are expressed as a percentage of the actual realization, both criteria have the advantage of being easy to interpret. Both are based on the forecast error, e_t . They are calculated using the following formulas³⁴:

$$MAPE = \frac{100}{n} \sum_{t=1}^n \frac{|e_t|}{y_t} \quad (2.32)$$

³²This technique relies on Wong et al. (2007).

³³The forecasting performance can be calculated using any other arbitrary fixed period.

³⁴See Witt and Witt (1991).

$$RMSE = \sqrt{\frac{100}{n} \sum_{t=1}^n \left(\frac{e_t}{y_t}\right)^2}. \quad (2.33)$$

According to Chen et al. (2009), percentages below 10% reflect high forecasting accuracy.

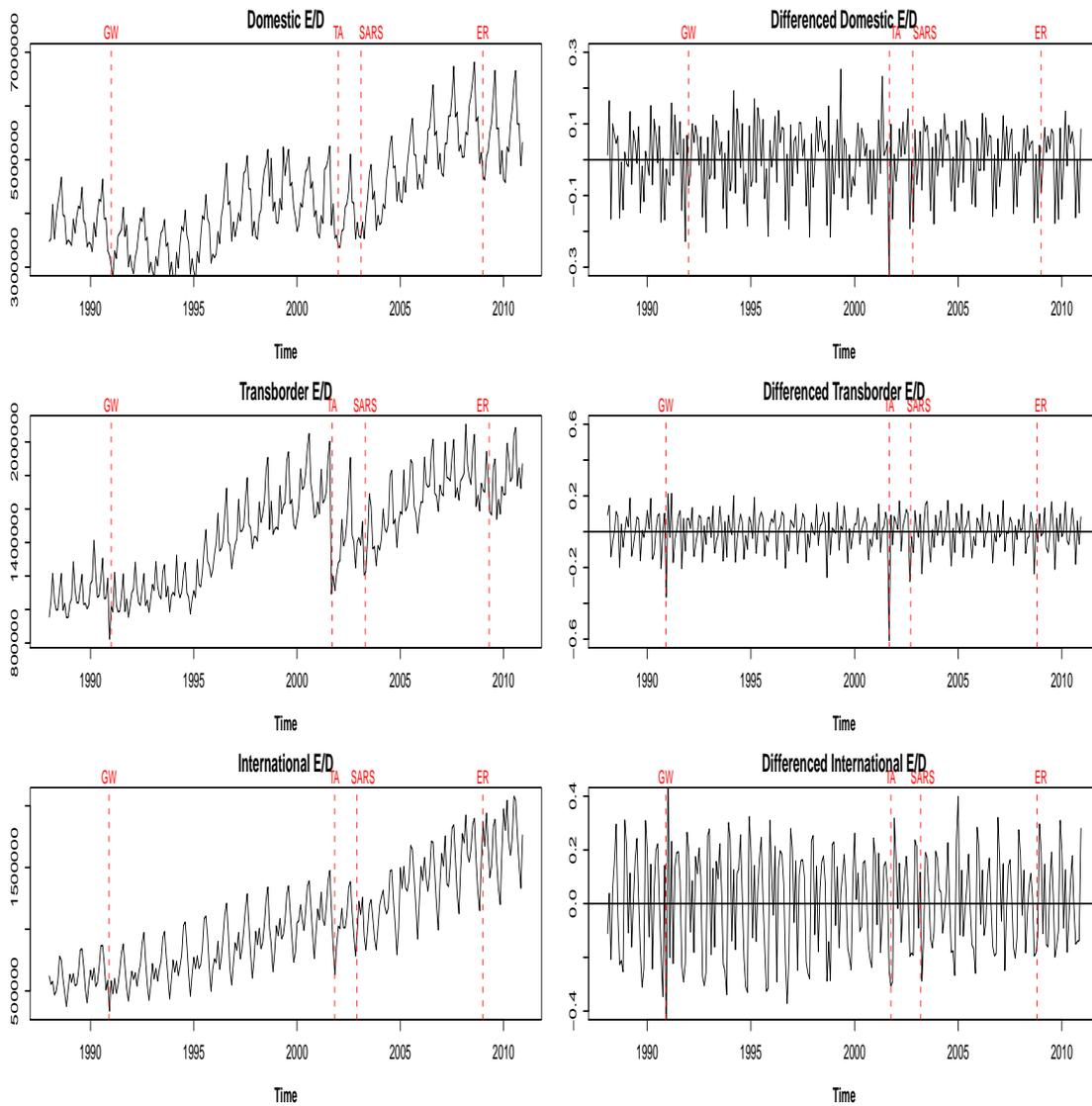
Chapter 3

Data

The data used in this paper are the monthly series of enplaned/deplaned air passengers for the domestic, transborder and international sectors for the period ranging from January 1988 to December 2010. These market segments are categories often employed and well identified in the air industry and they also correspond to those investigated in Emiray and Rodriguez (2003). The domestic sector encompasses all travel between two airports in Canada; the transborder sector includes all travel between an airport in the United States and an airport in Canada whereas the international sector considers the flights from/to a foreign country other than the United States and an airport in Canada (Emiray and Rodriguez, 2003). Our time series data are divided in two subsets, as done by Emiray and Rodriguez (2003). The first sample includes data from January 1988 to December 2008. It corresponds to our "**training data**", i.e the series used to fit our models. The second sample covers the period from January 2009 to December 2010 and it's used as "**evaluation data**" for measuring the forecasting performance.

Figure 3.1 shows the monthly number of enplaned/deplaned air passengers in Canada for all three sectors, as well as the first differenced log-transformed series for the entire sampling period. The three series share similar characteristics. First, the number of enplaned/deplaned air passengers in Canada is generally increasing over time, suggesting the presence of a positive time trend. Second, the variance of all three series is increasing over time, hinting at variance non-stationarity. Third, seasonality is clearly present in all series. The peaks correspond to the summer months while the troughs occur during the months of October and November. Moreover, the graphs show the location of four important events for the air transportation industry: the first Gulf war of 1991 (GW), the terrorist attacks of September 2001 (TA), the Severe Acute Respiratory Syndrome epidemic of 2002-2003 (SARS) and the economic recession of 2009 (ER). The first two events (GW and TA) are clearly followed by a negative impact on the series. Finally, we note that the first differenced log-transformed series of all three sectors seem to have stable mean and variance.

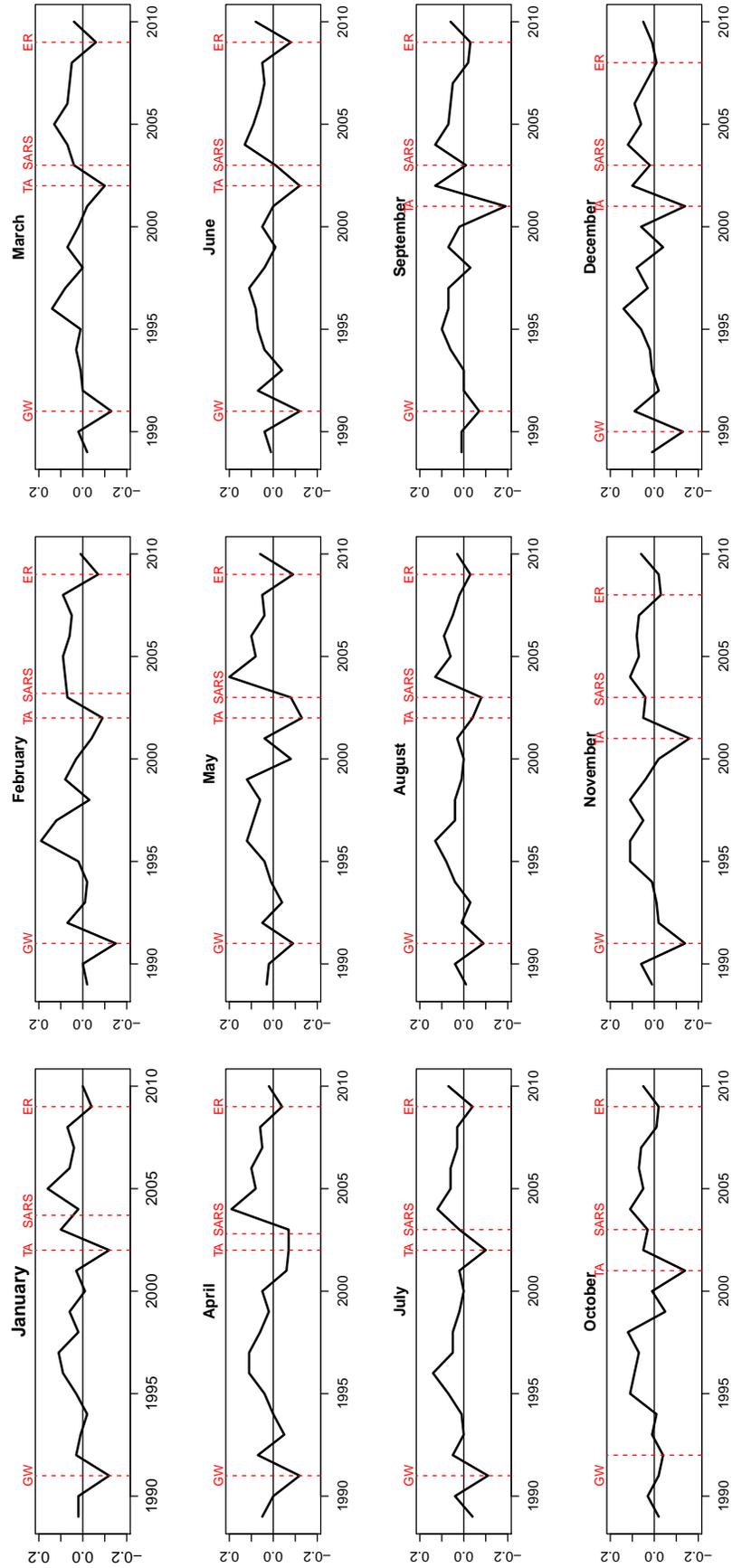
Figure 3.1: Canadian E/D air passenger series in levels and first differences



GW: Gulf war, TA: Terrorist attacks, ER: Economic recession, SARS : Severe Acute Respiratory Syndrom pandemic

Figure 3.2 shows the monthly growth rates of total enplaned/deplaned air passengers in Canada over the entire sampling period. This figure confirms and complements some of the results obtained previously. The dotted vertical lines show the following events: the First Gulf War of 1991 (GW), the 9/11 terrorist attacks in 2001 (TA), the 2009 economic recession (ER) and the SARS epidemic that outbreake between Novembre 2002 and July 2003.

Figure 3.2: Annual growth rates of air transport demand in Canada



GW: Gulf war, TA: Terrorist attacks, ER: Economic recession, SARS : Severe Acute Respiratory Syndrom pandemic

Chapter 4

Results

4.1 Single model estimations

The following section presents the estimation results of the four models described in sections 2.1 to 2.3. In addition, it evaluates and compares their forecasting performance.

4.1.1 Harmonic regression model

We begin by presenting in table 4.1 the final harmonic regression model selected after applying a backward selection procedure. The results of the Breush-Pagan and Durbin-Watson tests for heteroscedasticity and autocorrelation are also shown. These tests indicate the presence of heteroscedasticity and autocorrelation in the final models for all sectors (the lone exception is the international sector for which the null hypothesis of homoscedasticity of the residuals cannot be rejected at the 5% significance level). Therefore, the `vcovHAC()` function of the `sandwich` package in R was applied to get heteroscedasticity and autocorrelation robust covariance matrices. Note that the harmonic regression model includes dummy variables that account for several structural shocks that occurred during the sampling period : the first Gulf War (DGW), the terrorist attacks of September 2001 (D11S) and the SARS epidemic (DSARS).

$$DGW = \begin{cases} 1 & \text{for } t = 1990(11) \text{ to } 1991(2) \\ 0 & \text{otherwise} \end{cases}$$

$$D11S = \begin{cases} 1 & \text{if } t = 2001(09) \\ 0 & \text{otherwise} \end{cases}$$

$$DSARS = \begin{cases} 1 & \text{for } t = 2002(11) \text{ to } 2003(07) \\ 0 & \text{otherwise} \end{cases}$$

We observe that most regressors are significantly different from zero at the 5% level. The only exceptions are the variables COS[,5] and COS[,6] for the domestic sector. Regarding the structural shocks, the SARS epidemic is significant at the 0.1% level in all three sectors. As expected, its effect is negative¹. Canada was among the 26 countries affected by the epidemic, as evidenced by the travel warning to Toronto issued by the World Health Organisation (WHO) on the 23rd of April 2003 (Svoboda et al., 2004). This warning only lasted six days but cost the Toronto tourist industry a reported 260 million dollars (Svoboda et al., 2004).

Table 4.1: Harmonic regression model: estimation results

	Log (domestic)	Log(transborder)	Log(international)
Intercept	15.1723***	14.1796***	13.7287***
Time	0.1816***	0.2153***	0.3203***
(Time) ²	0.0604***	-0.0304***	-
(Time) ³	-0.0254***	-0.0208**	-
SIN[,1]	-0.0189**	0.0338***	0.0543***
COS[,1]	-0.1346***	-0.0539***	-0.1364***
SIN[,2]	0.0136*	0.0605***	0.1414***
COS[,2]	0.0134*	0.0288***	0.1315***
SIN[,3]	-	-0.0245**	-0.0269***
COS[,3]	-	-0.0365***	0.0179*
SIN[,4]	-	-	-0.0403***
COS[,4]	-	-	0.0199**
SIN[,5]	-0.0485***	-0.0395***	-0.0432***
COS[,5]	0.0098	0.0197*	0.0204**
SIN[,6]	-	-	-
COS[,6]	-0.0085.	-	0.0124*
DGW	-	-	-
D11S	-0.1689*	-0.3328***	-
DSARS	-0.1410***	-0.1692***	-0.1120***
BP-pvalues	0.0000	0.0002	0.4028
DW-pvalues	0.0000	0.0000	0.0000
R ²	0.8677	0.8321	0.9503
R ² _{adj}	0.8611	0.8229	0.9476

***, **, * and . denote significance at the 0.0001, 0.01, 0.05 and 0.1 level respectively. - denotes regressors which have been dropped in the backward selection procedure. BP-pvalues denotes the Breush-Pagan test p-values. DW-pvalues denotes the Durbin-Watson test p-values.

¹Recall that SARS was a world pandemic that affected 26 countries and led to the death of 774 individuals (Wilder-Smith, 2006).

Turning our attention to the terrorist attacks of September 11th 2001, the results in table 4.1 indicate that they had a significant and negative impact on air travel for the domestic and transborder sectors. However, note that this variable was dropped in the international sector model. This is surprising considering that this event led to higher fares and made air travel more difficult throughout the world (for example, through tighter security). Equally surprising is the fact that the dummy variable accounting for the Gulf War was dropped in all three models.

4.1.2 Holt-Winters exponential smoothing model

Let's now focus on the Holt-Winters exponential smoothing model. Firstly, figure 3.1 (in chapter 3) clearly shows that all three series possess a seasonal pattern with increasing variance. This points toward using a multiplicative model. Regarding the various smoothing parameters, the results for all three sectors are presented in table 4.2.

Table 4.2: Holt-Winters exponential smoothing model results

Sector	Type of seasonality	α	β	γ
Domestic	Multiplicative	0.4092	0.0114	0.2553
Transborder	Multiplicative	0.6292	0	0.3866
International	Multiplicative	0.3243	0	0.6502

We notice that the trend smoothing parameter (β) is either equal or very close to zero for all three sectors. This suggests that the slope of all three series is constant (or relatively constant) over time. Regarding the seasonal smoothing parameter (γ), the one of the international sector is the closest to one, which indicates that its seasonal pattern changes more over time than the one of the other sectors. This is not surprising, given that the international sector is expected to display a less systematic pattern as compared to the transborder or international markets. Finally, we should also mention that the level smoothing parameter (α) takes a higher value for the transborder sector, implying that the latter series is less steady than the others. While this result may seem surprising, the reality is that the United States was the country that was most directly affected by the terrorist attacks and the economic recession of 2009. In addition, the US were an important player in the First Gulf War. Clearly, all these events contributed to make the transborder series less steady.

4.1.3 Univariate time series models

We will now discuss the ARIMA and SARIMA models. We use the Box-Jenkins methodology to determine the appropriate specifications. The first step requires to verify that our series are variance and mean stationary. Figure 3.1 in chapter 3 shows that the logarithmic transformation needs to be applied to the series of all three sectors, given that all three display increasing variance over time. Mean stationarity is verified using the Augmented Dickey-Fuller test (ADF), whose results are presented in table 4.3. Note that the ADF test strategy is applied to the natural logarithm of the emplaned/deplaned

air passenger series. First, using equation (2.17a) from section 2.3.4, we test the null hypothesis of the presence of a unit root for all three market segments (domestic, transborder and international). The relevant empirical statistic τ_3 is above its 1% critical value (τ_3^*) in all three cases. Thus, the null hypothesis of a unit root cannot be rejected. Consequently, we test the joint hypothesis of the presence of a unit root without trend. The relevant empirical statistic is ϕ_2 , which is below its 1% critical value (ϕ_2^*) independently of the market segment considered. Therefore, the joint null hypothesis of a unit root with no trend is accepted. Hence, we use equation (2.17b) to test again for the presence of a unit root dropping the deterministic trend. The empirical statistic τ_2 being above its critical 1% cutoff τ_2^* , the test confirms that the log-level series have a unit root and need first differencing. We proceed by testing the 1st differenced series. The relevant empirical statistic is $\tilde{\tau}_2$, which is below its 1% critical value ($\tilde{\tau}_2^*$) for the transborder and international market segments and below its 5% critical value² for the domestic sector. Thus, the null hypothesis of the presence of a unit root can be rejected. Consequently, first-differencing is sufficient for obtaining stationary series.

Table 4.3: ADF test results

	Eq. 15a				Eq. 15b		Eq. 15b on 1st diff.	
	Empirical		Theoretical (1%)		Empirical	Theoretical(1%)	Empirical	Theoretical(1%)
	τ_3	ϕ_2	τ_3^*	ϕ_2^*	τ_2	τ_2^*	$\tilde{\tau}_2$	$\tilde{\tau}_2^*$
log(Domestic)	-2.43	3.44	-3.98	8.34	-0.57	-3.44	-3.20	-3.44
log(Transborder)	-2.35	2.78	-3.98	8.34	-1.17	-3.44	-4.15	-3.44
log(International)	-2.36	2.92	-3.98	8.34	0.11	-3.44	-6.09	-3.44

Subsequently, the Osborn, Chui, Smith and Birchenhall test (OCSB) is performed in order to determine whether or not seasonal differencing is required in the case of the SARIMA models. The results of this test suggest that seasonal differencing is not required. This implies that the seasonal pattern of all three series does not significantly change over time. It is important to note that this result does not necessarily contradict the one obtained with the Holt-Winters model, where the seasonal smoothing parameter suggested a seasonal pattern that changes over time for the international sector. On the contrary, the OSCB test can be seen as complementing the latter result by stating that the seasonal pattern does not sufficiently change over time to warrant seasonal differencing.

After determining the differencing parameters, we proceed by selecting the autoregressive and moving average orders using the AIC information criterion. The selected models for each sector are presented in table 4.4. This table also presents the results of the Ljung-Box test. This test is applied to verify the statistical validity of the selected models by checking for the presence of serial correlation in the residuals using a maximum lag order of 36 months (i.e. three years).

²The 5% critical value ($\tilde{\tau}_2^*$) is equal to -2.87

Table 4.4: ARIMA and SARIMA model selection results

Log(Domestic)	
ARIMA (3,1,2)	SARIMA(0,1,1)(2,0,0)₁₂
Ljung-Box (p-value):0.2116	Ljung-Box (p-value):0.03582
AIC: -976.59	AIC:-902.64
Log(Transborder)	
ARIMA (0,1,4)	SARIMA(2,1,1)(1,0,1)₁₂
Ljung-Box (p-value):0.09918	Ljung-Box (p-value):0.8445
AIC:-796.98	AIC:-773.76
Log(International)	
ARIMA(4,1,6)	SARIMA(2,1,1)(2,0,0)₁₂
Ljung-Box (p-value):0.9065	Ljung-Box (p-value):0.2487
AIC:-607.29	AIC:-540.47

The results of table 4.4 allow us to conclude that serial correlation is not present at the 5% significance level for all models, with the exception of the SARIMA model for the domestic sector. In the latter case, we cannot reject the null hypothesis of no serial correlation at the 1% significance level.

4.1.4 Forecasting performance of individual models

We conclude section 4.1 by comparing the forecasting accuracy of the single models using the MAPE and RMPSE performance criteria. The latter are calculated over the two year period of 2009-2010, which represents the evaluation period used in this work. Table 4.5 presents the results. First, all models provide accurate forecasts as all MAPE and RMPSE are lower than 10%, with only one value slightly over this threshold. The Holt-Winters and harmonic regression models are those with the poorest out-of-sample predictive performance, while ARIMA and SARIMA models provide the most accurate forecasts in general. The predictive performance of each model depends on the sector considered. The ARIMA specification is the best for predicting the domestic sector series while the SARIMA and Holt-Winters models are the most accurate for the transborder and international sectors respectively. This result is in line with Shen et al. (2011), who found that no single model systematically outperforms all others. Table 4.5 also clearly highlights that the same performance ranking holds with both performance criteria. This is not surprising, considering that both criteria are highly similar in nature.

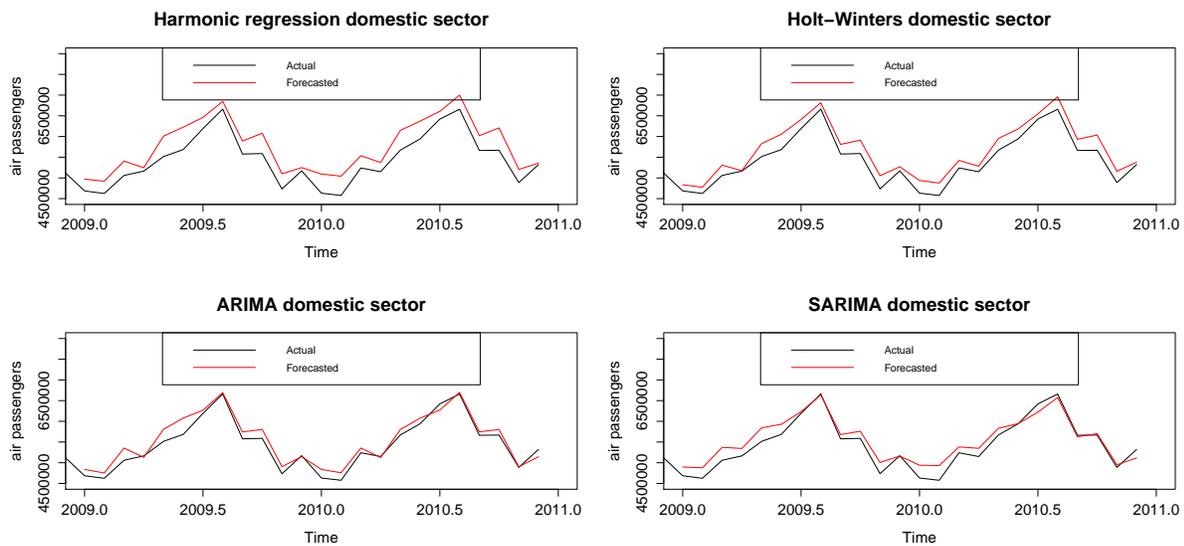
Table 4.5: MAPE and RMSPE for single models

	Harmonic model	Holt-Winters	ARIMA	SARIMA
MAPE				
Domestic	6.13 (4)	4.21 (3)	2.64 (1)	3.18 (2)
Transborder	6.65 (3)	7.36 (4)	6.23 (2)	6.11 (1)
International	9.49 (4)	3.92 (1)	6.61 (3)	4.29 (2)
RMSPE				
Domestic	6.71 (4)	4.59 (3)	3.18 (1)	3.89 (2)
Transborder	7.95 (3)	9.17 (4)	7.64 (2)	7.51 (1)
International	10.42 (4)	4.64 (1)	7.66 (3)	5.58 (2)

() denotes the rank of the model

Figures 4.1 to 4.3 show the actual and forecasted numbers of enplaned\deplaned air passengers in Canada for years 2009 and 2010. These graphs clearly demonstrate that all single models deliver a fair representation of the time series under investigation, capturing both their seasonality and level.

Figure 4.1: Actual and forecasted Canadian E/D air passenger numbers for the domestic sector



4.2 Combination forecasting techniques

Having discussed the forecasting performance of each individual model, we now examine the results obtained from combining the single model forecasts. This comparison is done over year 2010

Figure 4.2: Actual and forecasted Canadian E/D air passenger numbers for the transborder sector

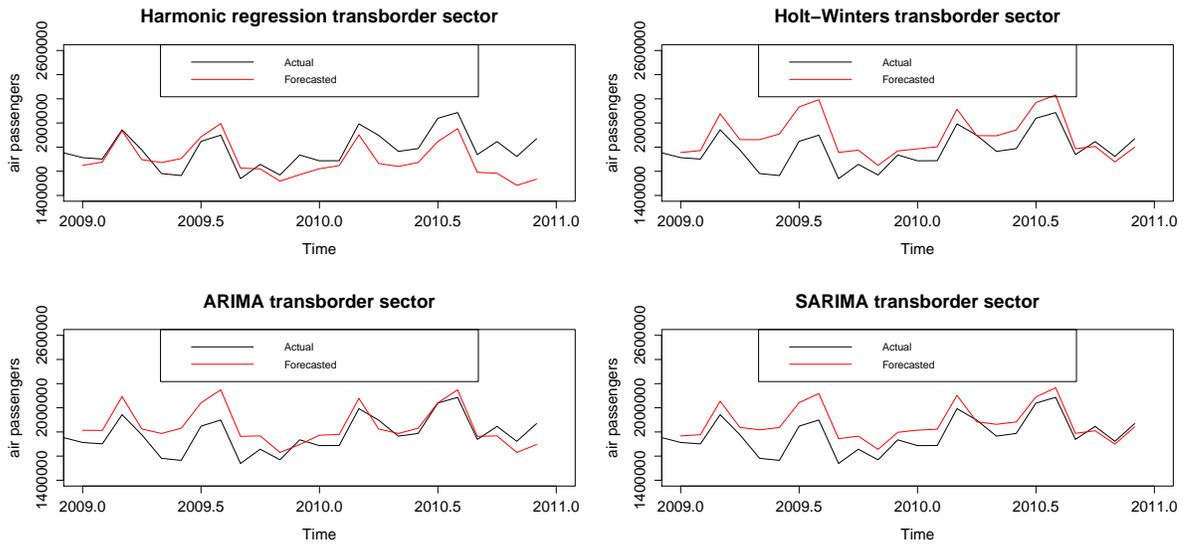
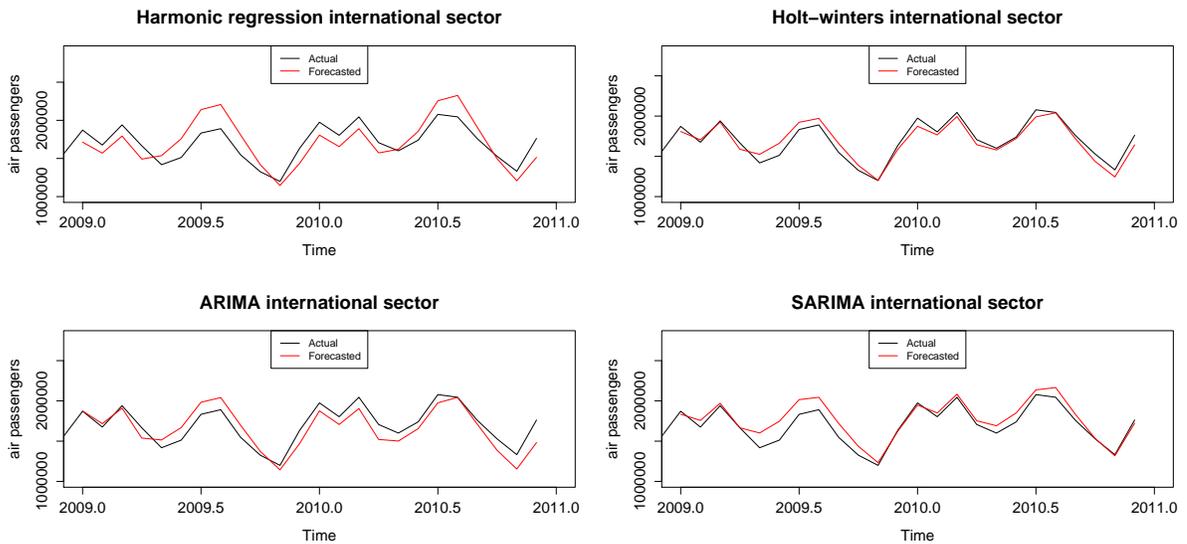


Figure 4.3: Actual and forecasted Canadian E/D air passenger numbers for the international sector



only and the evaluation dataset is adapted accordingly. The reason behind this is simple. Recall that the combination forecasts in the variance-covariance combination method take into consideration the past out-of-sample performance of each individual model being combined. This past performance is reflected in the combination weights, which are obtained using out-of-sample forecast errors. Computing these errors requires not only the actual value of the evaluation set, but also its out-of-sample forecast. We possess both quantities for every month of years 2009 and 2010. Consequently, we can compute 24 forecast errors. However, no out-of-sample forecast errors are available for the years before 2008, as these points are our training data. We were reluctant to compute weights (and by ex-

tension combination forecasts) for year 2009 based on in-sample predictions. This holds for all three versions of the variance-covariance method. Note that the simple average combination method allows us to compute combination forecasts for both 2009 and 2010. However, in order to keep identical sample sizes across combination methods, the out-of-sample predictive performances for the simple average method are based on projections made for year 2010 exclusively.

To keep our analysis tractable, we did not combine ARIMA and SARIMA forecasts since these are highly similar models (both are ARMA-based models) whose forecasts are likely to be correlated. Neither did we combine harmonic regression and Holt-Winters forecasts given that both are non-autoregressive models. Finally, the SARIMA models dominating in general the ARIMA ones, we restricted our analysis to combining SARIMA forecasts with those obtained from harmonic regressions and Holt-Winters models, in tables 4.6 and 4.7 respectively.

We begin by comparing the forecasting accuracy of the harmonic regression and SARIMA models as well as their combined version. Results are presented in table 4.6. We observe that whatever the combination technique and forecasting performance metric, combined forecasts are always more accurate than those obtained from the single worst model. This result holds across all market segments considered. This result slightly differs from the one presented by Wong et al. (2007). These authors find that there exist a few rare instances where the single worst model predictions are more accurate than the combined forecasts.

Furthermore, we notice that the combined forecasts outperform their single best model counterpart 3 times (out of 16) with each performance criterion. Most of these cases apply to the transborder sector and are displayed in bold characters in table 4.6. This suggests that the flight sector considered plays a role in whether or not a combined forecast manages to surpass its single model counterparts. The latter also likely depends on the combination technique used. Indeed, we observe that combined forecasts obtained through the seasonal variance-covariance method never outperform their single best model counterparts.

Table 4.6: Comparison of the forecasting accuracy of single and combined forecasts for Harmonic and SARIMA models (evaluation period: 2010(1)-2010(12)): MAPE and RMSPE

	Single model forecasts		Forecast combinations			
	Harmonic	SARIMA	SA	SVC	SWVC	FWVC
Domestic	6.40 (6.99)	2.88 (3.69)	4.19 (4.86)	3.61 (4.39)	3.35 (4.14)	2.90 (3.70)
Transborder	8.91 (10.0)	3.89 (4.55)	3.82 (4.94)	5.88 (6.72)	2.99 (3.77)	3.49 (3.85)
International	8.04 (8.81)	2.50 (3.00)	4.52 (4.91)	2.61 (3.06)	2.99 (3.54)	2.50 (2.99)

The RMSPE counterpart of the MAPE is in parenthesis. **BOLD** denotes that the combination forecast outperforms the individual forecasts. **SA**: Simple average method. **SVC**: Seasonal variance-covariance method. **SWVC**: variance-covariance method with shifting window. **FWVC**: variance-covariance method with fixed window.

When comparing the forecasting performance of the various combination methods in table 4.6, we

observe that the simple average combination method is usually the least accurate (the lone exception is the transborder sector, for which it is the second least accurate). This result is in line with the ones found in the tourism forecasting literature (see Coshall (2009), Wong et al. (2007), Shen et al. (2011) and Winkler and Makridakis (1983)). Finally, looking at table 4.6, we can also add that the fixed and shifting window variance-covariance methods are almost always more accurate than the seasonal one.

We proceed by comparing the forecasting accuracy of the Holt-Winters and SARIMA models. Results are presented in table 4.7. This table confirms some of the results previously obtained. Once again, we notice that combination forecasts always dominate their single worst model counterpart. This result is independent of the flight sector, the performance criterion and the combination technique used.

Table 4.7: Comparison of the forecasting accuracy of single and combined forecasts for Holt-Winters and SARIMA models (evaluation period: 2010(1)-2010(12)): MAPE and RMSPE

	Single model forecasts		Forecast combinations			
	H-W	SARIMA	SA	SVC	MWVC	FWVC
Domestic	4.39 (4.73)	2.88 (3.69)	3.22 (3.75)	3.14 (3.69)	3.13 (3.76)	2.88 (3.68)
Transborder	4.96 (5.54)	3.89 (4.55)	4.42 (4.96)	4.32 (4.81)	4.18 (4.76)	4.71 (5.26)
International	3.49 (4.11)	2.50 (3.00)	2.17 (2.60)	2.33 (2.92)	2.12 (2.38)	2.39 (2.76)

The RMSPE counterpart of the MAPE is in parenthesis. **BOLD** denotes that the combination forecast outperforms the individual forecasts. **SA**: Simple average method. **SVC**: Seasonal variance-covariance method. **SWVC**: variance-covariance method with shifting window. **FWVC**: variance-covariance method with fixed window.

We now observe that combined forecasts outperform their single best model counterpart more frequently than in the previous combination of models. This holds true 4 times (out of 16) when considering the MAPE criterion and 5 times (out of 16) when considering the RMPSE performance metric. The performance of our forecast combinations is slightly lower than the literature usually finds. Combined forecasts dominate more often those based on a single model, see for example Shen et al. (2011). A possible explanation for this can be found in Brown and Murphy (1996) and Wong et al. (2007). These authors suggest that forecast combinations are likely to achieve higher predictive performance when there is independence between the prediction errors of the models being combined. It may be that, in our case, the prediction errors of the combined models are less independent. But we lack of information from the literature to verify this hypothesis.

We also note that all the combined forecasts that are more accurate than their single best model counterpart using the MAPE criterion (as well as 4 out of 5 in the case of the RMPSE) are observed for the international sector. This confirms that whether or not combined forecasts outperform their best single model counterpart depends on the flight sector considered. Let us also observe that the performance (and ranking) of combination forecasts does not greatly vary no matter the performance indicator used (MAPE or RMPSE). This result is similar to the one found in Coshall (2009).

When considering the combination technique applied (SA, SVC, etc), we observe that the simple

average method offers either the worst or second worst performance for two out of three sectors considered.

Conclusion

This thesis tests the ability of several time-series models for predicting the monthly number of enplaned/deplaned air passengers in Canada for three market segments : domestic, transborder and international flights. Four standard models are explored: harmonic regression, Holt-Winters exponential smoothing, the autoregressive-integrated-moving average model (ARIMA) and the seasonal autoregressive-integrated-moving average model (SARIMA). We further test the performance of combining the single model predictions in different ways: by simply averaging the prediction of two single models or by combining their forecasts based on their past predictive performance (variance-covariance approach). Two global predictive accuracy metrics are used to carry out our analysis: MAPE (mean absolute percentage error) and RMSE (root mean squared error).

We first estimate our models on training data covering the period from January 1998 to December 2008. We then offer monthly predictions over the months of January 2009 to December 2010. Our results show that all single models provide highly accurate forecasts, below the 10% threshold of the pointwise realization on average. ARIMA and SARIMA model forecasts appear to always outperform harmonic regression and they are in general superior to Holt-Winters exponential smoothing in terms of out-of-sample performance, whatever the accuracy measure used and the market segment. Regarding the forecast combinations, we find that they are always more accurate than their single worst model counterpart. In addition, in some instances, they outperform the best single model forecasts. This depends on: (i) the flight sector considered, (ii) the single models being combined and (iii) the combination technique being applied. Indeed, we find that simple averaging is usually the least accurate forecast combination technique, which is in accordance with what is found in the literature.

Combination methods seem therefore to achieve their goal of improving the forecasting accuracy when applied to air passenger traffic series. This is of great interest for short and medium run planification in the airport industry. As mentioned by Wong et al. (2007), day-to-day business may not always allow the stylized exercise of finding the best model. However, relying on a combination of forecasts coming from different models should clearly reduce the risk of using the worst model. In addition, combining complementary models, capturing a variety of features of the data (such as non-autoregressive ones, short and long run memory, non-linearities) as it has been done in other countries, could contribute to better outline the specificities of the Canadian market.

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